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Expansion of Multicharged Plasma Clouds into Ionospheric Plasma with Magnetic Field

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Collisionless plasma cloud expansion into an extremely rarefied ionized medium is considered in the presence of a magnetic field when the medium is in motion and the cloud is retarded because of the electromagnetic interactions. The collisionless flow is computed on the basis of a hybrid model, in which the ionic motions are considered by means of kinetic equations and the electrons are governed by a hydrodynamic equation. The hybrid model of collisionless interaction between multicharged plasma flows in a magnetic field is presented.

Nomenclature

CGS_q	= electric charge in cgs units, $g^{1/2}cm^{3/2}s^{-1}$
c	= velocity of light, cm/s
$\boldsymbol{E}(E_r, E_\varphi, 0)$	= intensity of electrical field, $CGS_q \cdot cm^{-2}$
e	= charge of electrons, CGS_q
$f_k = f_k(\boldsymbol{r}, \boldsymbol{v}, t)$	= distribution function of ions of the grade k, $s \cdot cm^{-4}$
$\boldsymbol{H}(0, 0, H = H_z)$	= intensity of magnetic field, Oe
$\boldsymbol{J}$	= current density, $CGS_q \cdot cm^{-2} \cdot s$
M_A	= Alfven Mach number
m	= mass of a particle, g
m_p	= mass of the pth quasi particle
N_k	= number of the particle grades
N_p	= total number of quasi particles
n	= concentration, cm^{-3}
q	= charge of a particle, CGS_q
R_H	= Larmor radius
R_{max}	= radial boundary of the calculated domain
R_0	= initial radius of the plasma cloud, cm
$\boldsymbol{r}$	= spatial coordinate, cm
r	= radial coordinate
r_p	= radial coordinate of the pth quasi particle
t	= time, s
$\boldsymbol{V}$	= average velocity, cm/s
V_A	= Alfven velocity, cm/s
V_0	= initial velocity of the plasma cloud expansion, cm/s
$\boldsymbol{v}$	= velocity of ions, cm/s
$v_{p,r}$	= radial projection of the velocity of the pth quasi particle
$v_{p,\varphi}$	= azimuthal projection of the velocity of the pth quasi particle
σ	= conductivity, s^{-1}

Subscripts

e	= electron
i	= ion
k	= grade of particles
$k = 1$	= ions of the plasma cloud with charge $q_1 = +e$
$k = 2$	= ions of the plasma cloud with charge $q_2 = +2e$ or $+e$
$k = 3$	= ions of the background plasma with charge $q_3 = +e$
p	= quasi particle
r, φ, z	= radial, azimuth, and axial coordinates
$*$	= undisturbed background plasma

Superscript

$\tilde{}$	= dimension parameters

Introduction

THE processes accompanying expansion of plasma clouds in a magnetic field in a vacuum or in a partially ionized environment attract attention in connection with a wide spectrum of the fundamental and applied physical applications, for example, in astrophysical research of near and distant space,[1] active experiments in space,[2] and laboratory modeling of laser plasma.[3] Development of representations about investigated processes has identified a number of urgent issues concerning the occurrence of large- and small-scale plasma instabilities, abnormal ionization of neutral rarefied environment, etc.

The first theoretical models of explosions in the ionosphere and shock-wave propagation in near-Earth space[1] initiated development of various computing models, in which both qualitative and quantitative descriptions of the phenomena were done. Magnetohydrodynamic (MHD) models have turned out to be convenient for the analysis of a general picture of the expansion process of plasma clouds, whereas kinetic–hydrodynamic (hybrid) models[4] have allowed the study of the structure of collisionless shock waves. A comparison of calculated results for the same problem about cylindrical expansion of plasma clouds in MHD and hybrid statements is carried out in Ref. 4. Recommendations for the choice of a specific heat ratio in much more economic MHD models are given. This specific heat ratio enables agreement with the results of hybrid calculations.

In the class of problems dealing with the structure of large-scale disturbances in the Earth's ionosphere, there remain many questions concerning possible development of plasma instabilities at the initial stage of the process, that is, until the expanding plasma is stopped by magnetic or gasdynamical pressure.

This paper studies the process of collisionless expansion of a cylindrical cloud of ionized gas in the presence of a magnetic field. The distinctive peculiarity of the given study is the investigation of multicharged expanding plasma dynamics.

Mathematical Model

For the description of collisionless dynamics of multicharged plasma, the hybrid model is used, in which electrons are considered as a liquid without mass, and the behavior of ions is simulated as a movement of collisionless clouds of quasi-ion particles.

A distribution function of ions allows the determination of the following averaged characteristics:

$$n_k(\boldsymbol{r}, t) = \int_0^\infty f_k \, dv, \qquad \boldsymbol{u}_k(\boldsymbol{r}, t) = \frac{1}{n_k} \int_0^\infty \boldsymbol{v} f_k \, dv \qquad (1)$$

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Total volumetric concentration of the ions, velocity, and current density is determined by the following formulas:

$$n(\boldsymbol{r},t)=\sum_{k}^{N_k} n_k(\boldsymbol{r},t),\qquad n\boldsymbol{V}(\boldsymbol{r},t)=\sum_{k}^{N_k} n_k\boldsymbol{u}_k(\boldsymbol{r},t) \tag{2}$$

$$\boldsymbol{J}(\boldsymbol{r},t)=\sum_{k}^{N_k} q_k\boldsymbol{u}_k(\boldsymbol{r},t)n_k(\boldsymbol{r},t)-en_e\boldsymbol{V}_e \tag{3}$$

The absence of any ionizing processes is supposed (by radiation or because of abnormal ionization, connected to heating of electrons at the collective plasma interactions). The Larmor radius of electrons $R_{H,e}$ is much smaller then the Larmor radius of ions $R_{H,i}$. The distribution function of electrons exhibits sharp changes on the small-scale $R_{H,e}$. It is appropriate to smooth the distribution function of electrons for the purposes of the present analysis, being interested only in a movement of the centers of their small-scale rotations. It can be made, considering the electrons as a liquid, on the basis of the hydrodynamic description of their movement with mass speed $\boldsymbol{V}_e$ and density n_e. Besides, we shall use the assumption about quasi neutrality of the plasma (at least on scales of the Larmor radius of ions). Therefore, with sufficient accuracy

$$en_e=\sum_{k}^{N_k} q_k n_k \tag{4}$$

If we neglect the electronic pressure caused by friction between electrons and ions (which is connected to collisions), the equation of electron movement can be written as follows:

$$m_e\frac{\partial\boldsymbol{V}_e}{\partial t}=-e\left(\boldsymbol{E}+\frac{\boldsymbol{V}_e\times\boldsymbol{H}}{c}\right) \tag{5}$$

This equation is the result of averaging the velocities from the similar equation for individual electrons, in which instead of the average velocity $\boldsymbol{V_e}$ there is the true velocity of electrons $\boldsymbol{v_e}$. The elimination from consideration of the small-scale rotations of electrons is equivalent to the assumption of zero Larmor radius, and consequently it is equivalent to the following assumption: $m_e=0$. The equation of motion of the electronic liquid in this approximation looks as follows:

$$\boldsymbol{E}+\frac{\boldsymbol{V}_e\times\boldsymbol{H}}{c}=0 \tag{6}$$

This corresponds to a force-free condition for the electrons. For the solution of the problem, one more approximation—about infinite conductivity—is used.

The approximations used to make the basis of the hybrid model of collisionless plasma used in the given work are as follows:

$$\frac{\partial f_k}{\partial t}+\boldsymbol{v}\frac{\partial f_k}{\partial\boldsymbol{r}}+\frac{q_k}{m_k}(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{H})\frac{\partial f_k}{\partial\boldsymbol{v}}=0 \tag{7}$$

$$\mathrm{rot}\boldsymbol{E}=-\frac{\partial\boldsymbol{H}}{\partial t} \tag{8}$$

$$\boldsymbol{E}+\boldsymbol{V}_e\times\boldsymbol{H}=0 \tag{9}$$

$$\mathrm{rot}\boldsymbol{H}=M_A^2\left(\sum_{k=1}^{N_k} q_k n_k\boldsymbol{u}_k-\boldsymbol{V}_e\sum_{k=1}^{N_k} q_k n_k\right) \tag{10}$$

where the following dimensionless parameters are used:

$$t=\tilde{t}\Omega_{H_*},\qquad r=\frac{\tilde{r}}{R_{H_*}},\qquad V=\frac{\tilde{V}}{V_0}$$

$$H=\frac{\tilde{H}}{H_*},\qquad E=\frac{\tilde{E}}{V_0H_*/c},\qquad m_k=\frac{\tilde{m}_k}{m_*}$$

$$M_A^2=4\pi\frac{n_*m_*V_0^2}{H_*^2}=\left(\frac{V_0}{V_A}\right)^2,\qquad \Omega_{H_*}=\frac{eH_*}{m_*c}$$

$$R_{H_*}=\frac{V_0}{\Omega_{H_*}},\qquad q_k=\frac{\tilde{q}_k}{e} \tag{11}$$

Numerical Simulation Method

The system of Eqs. (7–10) is solved by the particle method. However, instead of the kinetic Eq. (7) the following equations for quasi particles are solved:

$$\frac{\mathrm{d}r_p}{\mathrm{d}t}=v_{p,r},\qquad p=1,\,2,\,\ldots,\,N_p \tag{12}$$

$$\frac{\mathrm{d}v_{p,r}}{\mathrm{d}t}=\frac{v_{p,\varphi}^2}{r}+\frac{1}{m_p}(E_r+v_{p,\varphi}H),\qquad p=1,\,2,\,\ldots,\,N_p \tag{13}$$

$$\frac{\mathrm{d}v_{p,\varphi}}{\mathrm{d}t}=-\frac{v_{p,r}\cdot v_{p,\varphi}}{r_p}+\frac{1}{m_p}(E_\varphi-v_{p,r}H)$$

$$p=1,\,2,\,\ldots,\,N_p \tag{14}$$

To integrate the system of Eqs. (12–14), the finite difference scheme of the second order of accuracy is used. After Eqs. (12–14) are solved, average functions for each kth grade of particles and total functions (2) and (3) are calculated. Then for determination of magnetic H and electrical $E_r,\ E_\varphi$ fields, the following system of equations is solved:

$$\frac{\partial H}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}(rW_rH) \tag{15}$$

$$E_\varphi=HW_r \tag{16}$$

$$E_r=-W_\varphi H-\frac{1}{n_eM_A^2}H\frac{\partial H}{\partial r} \tag{17}$$

$W_r,\ W_\varphi$ are the radial and azimuth projection of the vector:

$$\boldsymbol{W}=\frac{\sum_k^{N_k} q_kn_k\boldsymbol{u}_k}{\sum_k^{N_k} q_kn_k} \tag{18}$$

Equation (15) is solved using the flux-correct transport (FCT) scheme to the second order of accuracy.[5]

Numerical Simulation Results

The problem is solved in the geometry, which is shown in Fig. 1. The plasma cloud and background plasma are homogeneous at time $t=0$. The following initial conditions were used.

At $r<R_0$:

$$n_0=8.3\cdot10^{10},\qquad V=V_0r/R_0,\qquad V_0=126\cdot10^6$$

$$H=H_0=0,\qquad N_p^1=4000$$

$$N_p^2=4000,\qquad m_1=m_2=27$$

At $r>R_0$:

$$n_0=0,\qquad v=0,\qquad H=H_*=0.5,\qquad n_*=0.96\cdot10^7$$

At $0<r<R_{\max}$:

$$m_3=16,\qquad N_p^3=8000$$

The following parameters of modeling correspond to these conditions:

$$M_A=16,\qquad \Omega_{H_*}=313\,\mathrm{s}^{-1},\qquad R_{H_*}=0.453\cdot10^6\,\mathrm{cm}$$

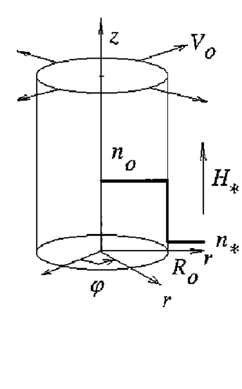

Fig. 1 Geometry of the problem.

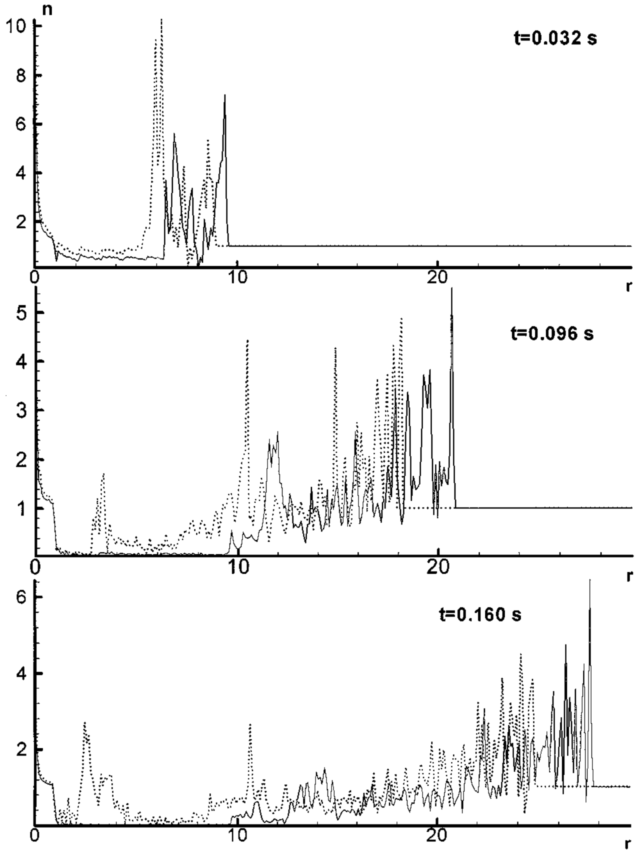

Fig. 2 Radial distribution of the total ionic concentrations at the sequential instants: ——, the expansion of the plasma cloud of singly ionized atoms, and – – –, the expansion of the plasma cloud consisting of singly and doubly charged ions.

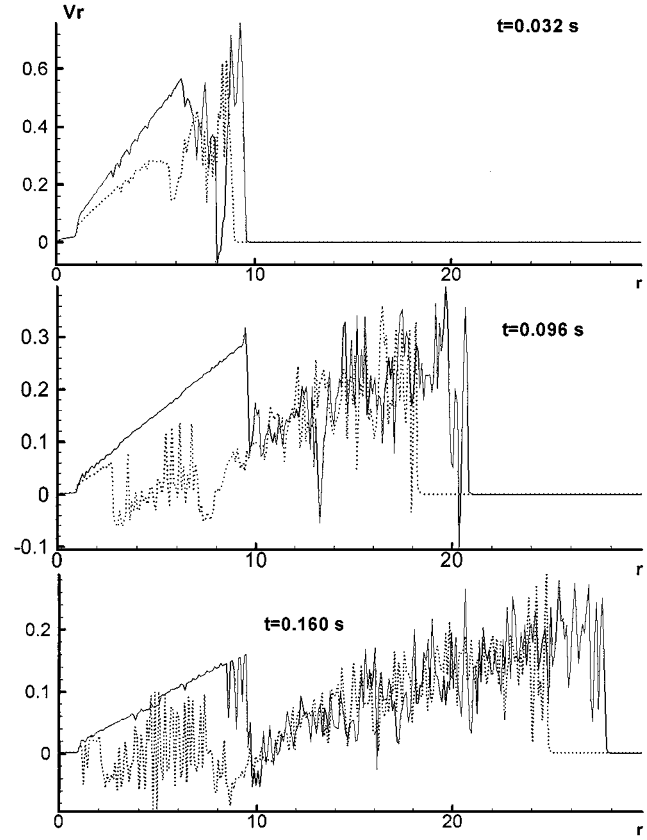

Fig. 3 Radial distribution of the total velocity *V* at the sequential instants: ——, the expansion of the plasma cloud of singly ionized atoms, and – – –, the expansion of the plasma cloud consisting of singly and doubly charged ions.

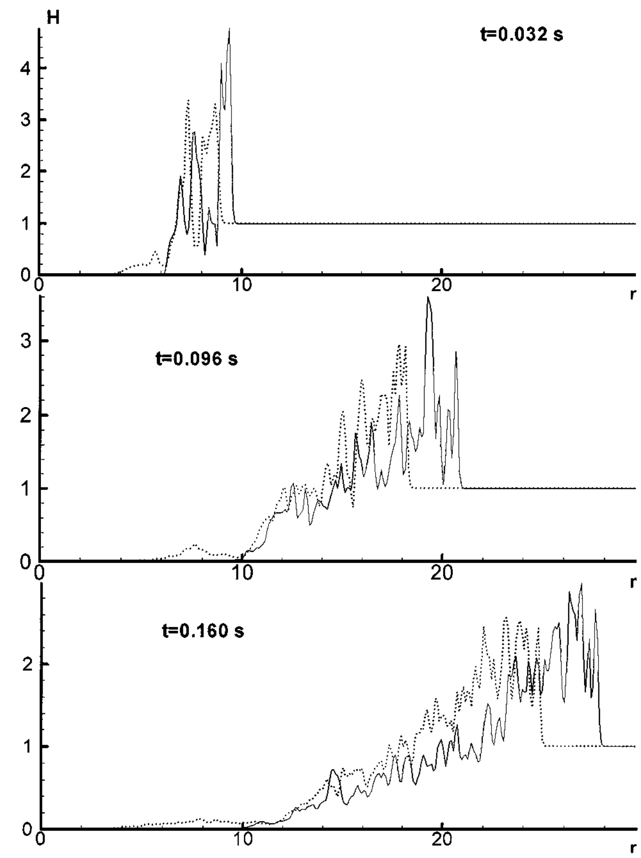

Fig. 4 Radial distribution of the magnetic field *H* at the sequential instants: ——, the expansion of the plasma cloud of singly ionized atoms, and – – –, the expansion of the plasma cloud consisting of singly and doubly charged ions.

The numerical simulation results for this variant are presented in Figs. 2–9. Each of Figs. 2–6 contains two curves. The first one corresponds to the expansion of the plasma cloud of singly ionized atoms ($q_1 = q_2 = e$). The second one corresponds to the expansion of the plasma cloud consisting of singly and doubly charged ions ($q_1 = e$, $q_2 = 2e$) with equal initial concentrations ($n_1 = n_2$).

The basic purpose of these calculations is to study the peculiarities of plasma flow structure, connected to the presence of multicharged ions in the plasma.

The interaction of the expanding plasma with the background plasma in the presence of a magnetic field has the following peculiarities (regardless of the charging structure of the plasma):

1) The expanding plasma creates a collisionless shock wave in the background plasma, which independently spreads in the radial direction (Figs. 2 and 3).

2) The propagation of the collisionless shock wave is accompanied by strong perturbations of the magnetic field (Fig. 4) and by generation of the vortex electrical field and the radial and azimuthal components of which are shown in Figs. 5 and 6.

3) On the border of the expanding plasma cloud, there is the area of a multiflow movement (Figs. 7 and 8).

4) The multiflow movement of ions is observed in the collisionless shock wave in the background plasma (Fig. 9). This is seen in the oscillating character of ion concentration (Fig. 2), average velocity (Fig. 3), magnetic field (Fig. 4), and electrical field (Figs. 5 and 6).

5) The magnetic field is superseded from the perturbed area, and local maxima of its intensity are observed at the boundary of the area (Fig. 4).

The process of the expansion of the doubly charged cloud of ions has the following peculiarities:

1) The external border of the doubly charged ion cloud is braked faster than the same singly charged ion cloud (Figs. 7 and 8).

2) The collisionless shock wave in the background plasma spreads with rather smaller speed (Fig. 3).

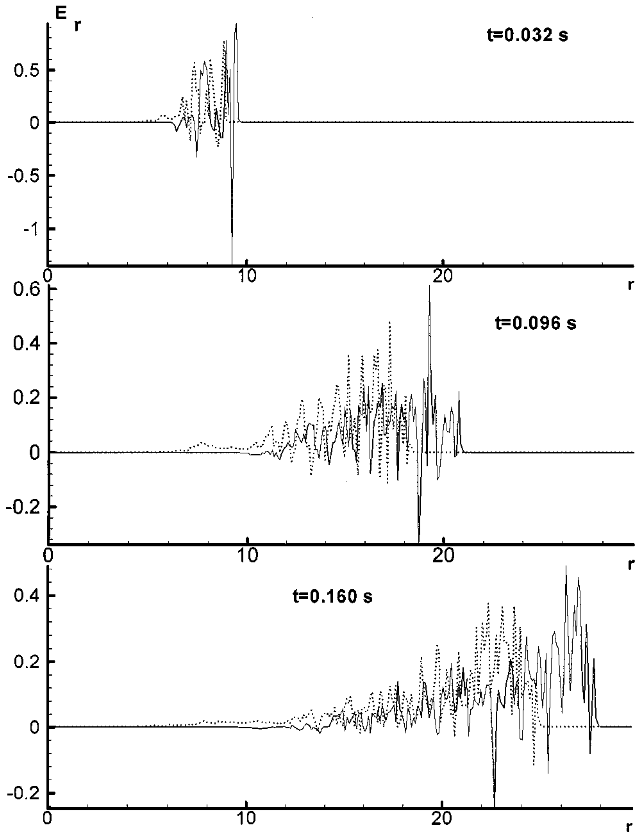

Fig. 5 Radial distribution of the radial component of the electric field at the sequential instants: ——, the expansion of the plasma cloud of singly ionized atoms, and –––, the expansion of the plasma cloud consisting of singly and doubly charged ions.

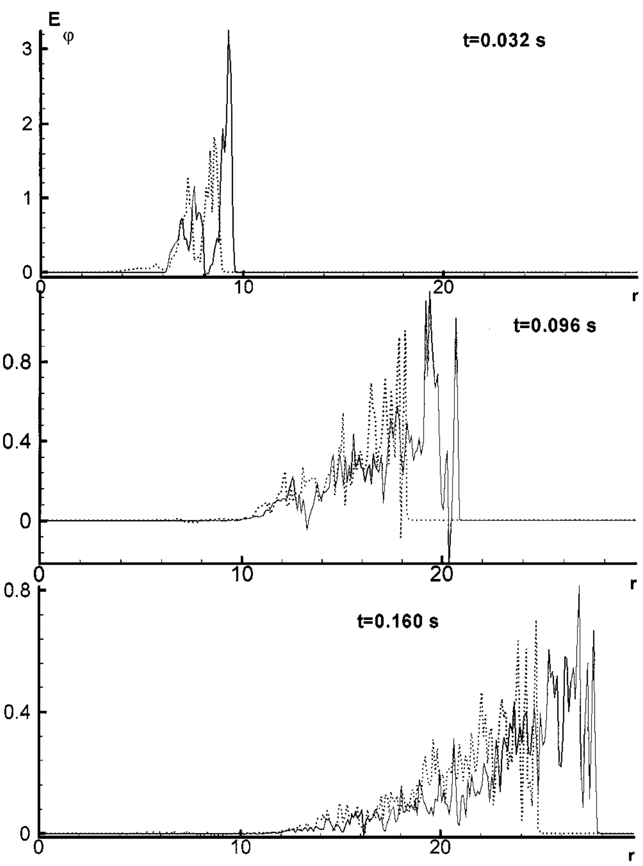

Fig. 6 Radial distribution of the azimuthal component of the electric field at the sequential instants: ——, the expansion of the plasma cloud of singly ionized atoms, and –––, the expansion of the plasma cloud consisting of singly and doubly charged ions.

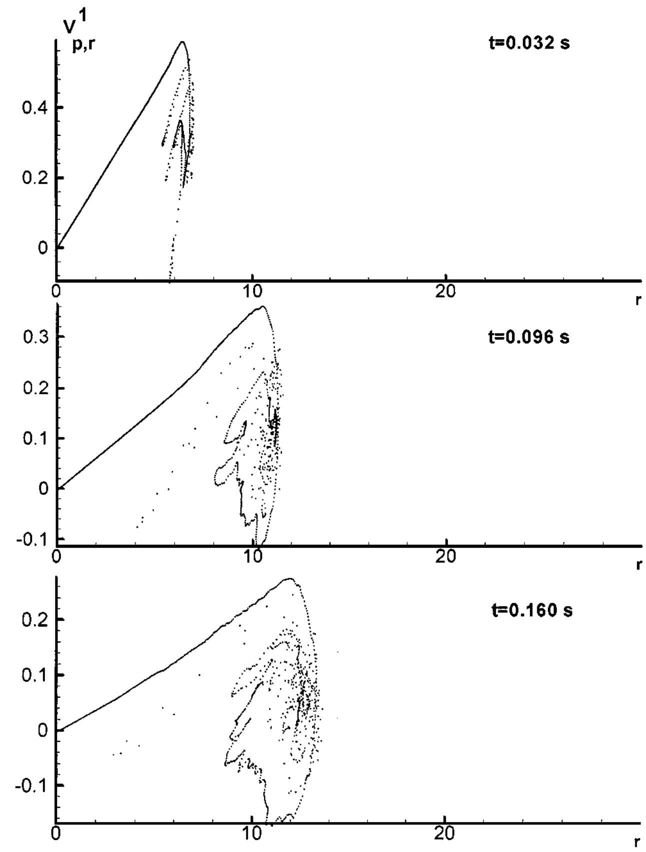

Fig. 7 Distribution of quasi particles of the first kind in the r-v_r phase space at the sequential instants (the expansion of the plasma cloud consisting of singly and doubly charged ions).

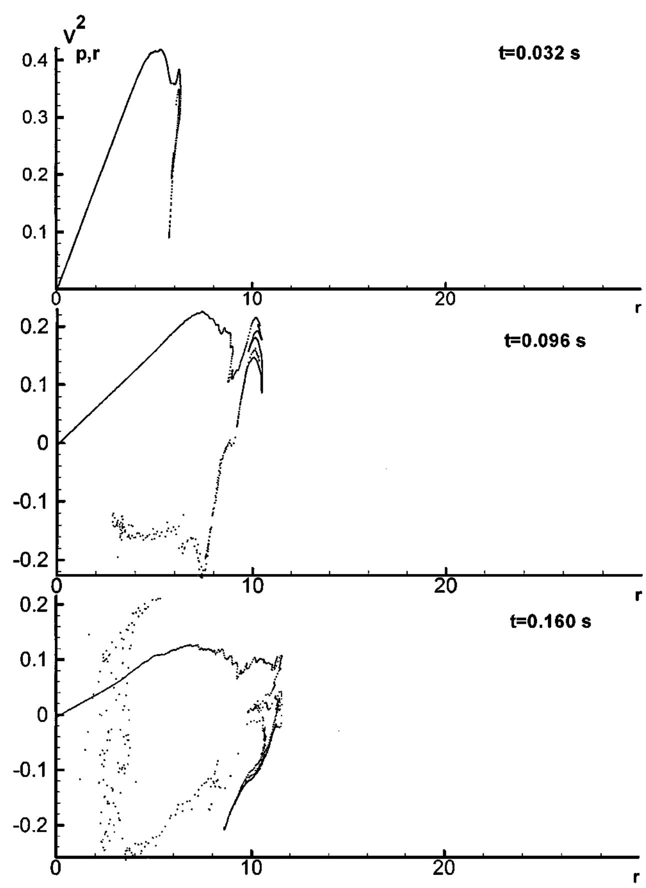

Fig. 8 Distribution of quasi particles of the second kind in the r-v_r phase space at the sequential instants (the expansion of the plasma cloud consisting of singly and doubly charged ions).

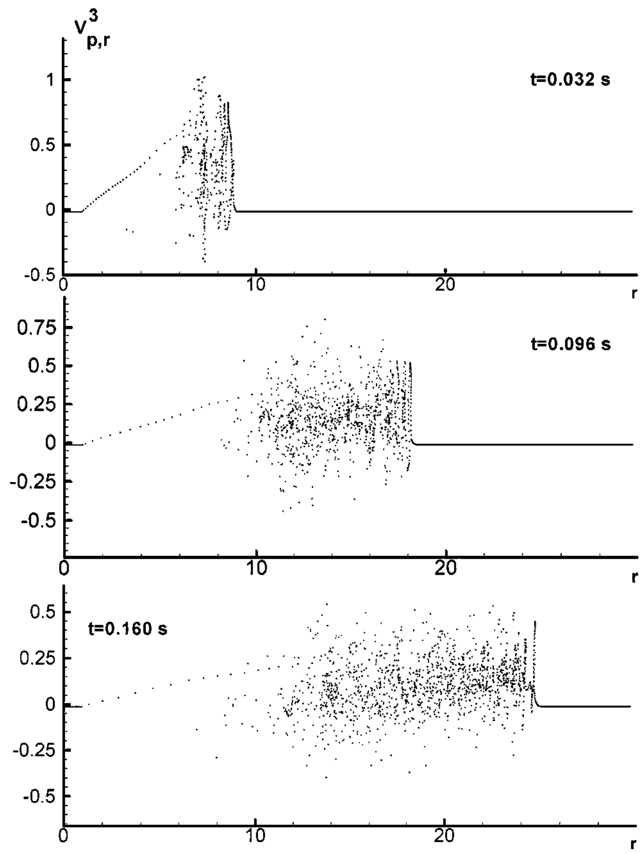

Fig. 9 Distribution of quasi particles of the third kind in the r-v_r phase space at the sequential instants (the expansion of the plasma cloud consisting of singly and doubly charged ions).

3) The doubly charged ions are essentially grouped in the internal regions of the perturbed area. The external environment of the plasma cloud consists essentially of singly charged ions (Fig. 7).

4) The radial movement of the singly and doubly charged ions has oscillating character (Figs. 7 and 8).

Conclusion

The hybrid (kinetic–hydrodynamic) model of collisionless interaction of the expansion plasma cloud with background plasma in a magnetic field is developed. The peculiarity of the model is the inclusion of doubly charged ions in the expanding plasma. The numerical simulation study shows that the presence of multicharged ions in expanding plasma clouds can result in appreciable change of structure of plasma flows in the perturbed area.

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